Effects of Chemical Potential on Hadron Masses in the Phase Transition Region *

QCD-TARO Collaboration: Ph. de Forcrand[a], M. García Pérez[b], T. Hashimoto[c], S. Hioki[d], Y. Liu[e] H. Matsufuru[j], O. Miyamura[e], A. Nakamura[g], I.-O. Stamatescu[f,h], T. Takaishi[i] and T. Umeda[e]

[a] ETH-Zürich, CH-8092 Zürich, Switzerland
[b] Dept. Física Teórica, Universidad Autónoma de Madrid, E-28049 Madrid, Spain
[c] Dept. of Appl. Phys., Fac. of Engineering, Fukui Univ., Fukui 910-8507, Japan
[d] Dept. of Physics, Tezukayama Univ., Nara 631-8501, Japan
[e] Dept. of Physics, Hiroshima Univ., Higashi-Hiroshima 739-8526, Japan
[f] Inst. Theor. Physik, Univ. of Heidelberg, D-69120 Heidelberg, Germany
[g] Res. Inst. for Inform. Sci. and Education, Hiroshima Univ., Higashi-Hiroshima 739-8521, Japan
[h] FEST, Schmeilweg 5, D-69118 Heidelberg, Germany
[i] Hiroshima University of Economics, Hiroshima 731-0192, Japan
[j] RCNP, Osaka University, Osaka 567, Japan

We study the response of hadron masses with respect to chemical potential at $\mu = 0$. Our preliminary results of the pion channel show that $\partial m/\partial\mu$ in the confinement phase is significantly larger than that in the deconfinement phase, which is consistent with the chiral restoration.

1. Introduction

As suggested by QCD sum rule analysis [1], hadron masses may be affected by density effects. This may explain some results of heavy ion collision experiments such as dilepton spectra and J/Ψ suppression.

It is difficult to introduce density effects in lattice QCD calculations due to the well-known "complex action" problem. Here we calculate the response of hadron masses to chemical potential, $\partial m/\partial\mu$, on dynamical configurations with $\mu = 0$. Since simulations are done at $\mu = 0$, there is no difficulty in obtaining $\partial m/\partial\mu$. We investigate the dependence of $\partial m/\partial\mu$ with the temperature.

2. Formulation

We use 2 flavors of staggered quarks. The effective action to simulate N_f fermion flavors is

$$S_{eff} = S_G + S_F \tag{1}$$

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where S_G is the standard plaquette action and

$$S_F = \frac{N_f}{4}\mathrm{Tr}\ln M(U,\mu) \tag{2}$$

where $M(U,\mu)$ is the staggered fermion Matrix.

The zero momentum hadron correlation function $G(t)$ is given by

$$G(t) = \sum_x < H(x,t)H(0,0)^\dagger > \tag{3}$$

and

$$\begin{aligned} &< H(x,t)H(0,0)^\dagger > \\ &= \int dU H(x,t)H(0,0)^\dagger \exp(-S_{eff})/Z \end{aligned} \tag{4}$$

where Z is the partition function.

Taking a derivative of the hadronic correlator with respect to μ.

$$\begin{aligned} \frac{\partial < H(x,t)H(0,0)^\dagger >}{\partial\mu} &= < \frac{\partial C(x,t)}{\partial\mu} > \\ - < C(x,t)\frac{\partial S_F}{\partial\mu} > &+ < C(x,t) >< \frac{\partial S_F}{\partial\mu} > . \end{aligned} \tag{5}$$

where $C(x,t)=H(x,t)H(0,0)^\dagger$. We calculate eq. (5) on dynamical configurations with $\mu=0$. In the case of $\mu=0$ eq. (5) can be simplified using the following facts:

(A) $\partial S_F/\partial\mu$ corresponds to the fermion number operator. Thus, the average of the fermion number operator at $\mu=0$ is zero: $<\frac{\partial S_F}{\partial\mu}>=0$.

(B) On each configuration the value of $\partial S_F/\partial\mu$ is purely imaginary [2]. Thus, the value of $<C(x,t)\frac{\partial S_F}{\partial\mu}>$ is also purely imaginary provided that the operator $C(x,t)$ is real. This is indeed the case if we consider $C(x,t)$ for mesons made up of degenerate quarks.

Using the facts (A) and (B) above we derive

$$\frac{\partial<H(x,t)H(0,0)^\dagger>}{\partial\mu}=<\frac{\partial C(x,t)}{\partial\mu}> \qquad (6)$$

for mesons consisting of degenerate quarks.

In the spectral representation,

$$G(t)=\sum_i A_i\cosh(m_i(t-N_t/2)). \qquad (7)$$

Taking a derivative of eq. (7) with respect to μ we obtain

$$\frac{\partial G(t)}{\partial\mu}=\sum_i[\frac{\partial A_i}{\partial\mu}\cosh(m_i(t-N_t/2)) \qquad (8)$$
$$+\frac{\partial m_i}{\partial\mu}A_i(t-N_t/2)\sinh(m_i(t-N_t/2))].$$

Our procedure to obtain $\partial m/\partial\mu$ is as follows. First we determine A_i and m_i by fitting correlation function data to eq. (7). Substituting the values of A_i and m_i into eq. (8) we fit the data of $\frac{\partial G(t)}{\partial\mu}$ to eq. (8). Then we obtain $\partial m_i/\partial\mu$ and $\partial A_i/\partial\mu$ as fitting parameters.

3. Definition of $\partial/\partial\mu$

We study the two flavor case (u and d quarks). In this case, we have two independent chemical potentials, μ_u and μ_d. Instead, the following combinations are convenient, $\mu_S=(\mu_u+\mu_d)/2$ and $\mu_V=(\mu_u-\mu_d)/2$ whith μ_S the usual chemical potential corresponding to baryon number. Then derivatives with respect to μ_S and μ_V are

$$\frac{\partial}{\partial\mu_S}=\frac{\partial}{\partial\mu_u}+\frac{\partial}{\partial\mu_d}=\frac{\partial}{\partial\mu_u}-\frac{\partial}{\partial\mu_{\bar d}} \qquad (9)$$

$$\frac{\partial}{\partial\mu_V}=\frac{\partial}{\partial\mu_u}-\frac{\partial}{\partial\mu_d}=\frac{\partial}{\partial\mu_u}+\frac{\partial}{\partial\mu_{\bar d}}\quad . \qquad (10)$$

For degenerate systems of u and d quarks,

$$\frac{\partial C_{u\bar d}}{\partial\mu_S}=\frac{\partial C_{u\bar d}}{\partial\mu_u}-\frac{\partial C_{u\bar d}}{\partial\mu_{\bar d}}=0\quad . \qquad (11)$$

at $\mu_u=\mu_d=0$. In this study we analize $\partial/\partial\mu_V$ which gives non-trivial results even with degenerate quarks. In the following $\partial/\partial\mu$ stands for $\partial/\partial\mu_V$.

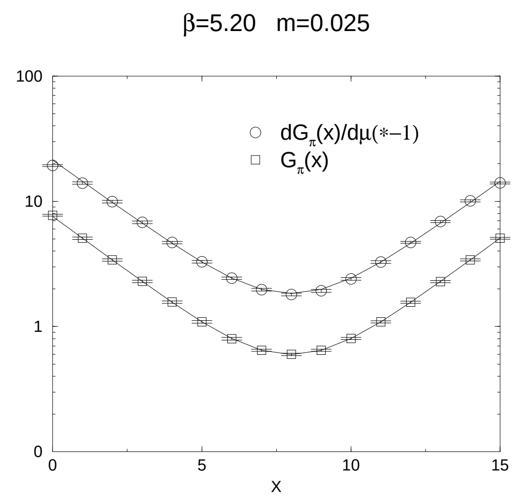

Figure 1. The pion correlation function, $G_\pi(x)$ and its derivative with respect to the chemical potential, $\frac{\partial G_\pi(x)}{\partial\mu}$ at $\beta=5.20$. $\frac{\partial G_\pi(x)}{\partial\mu}$ gives negative values. To plot them in logarithmic scale, they are multiplied by -1. Single pole fitting results are also shown, represented by solid lines.

4. Preliminary results

We present preliminary results of $\partial m/\partial\mu$ for $N_f=2$ staggered quarks. Simulations are done on a lattice of size $16\times8\times8\times4$ at $m_q=0.025$ with $\beta=5.20$, 5.26, 5.32 and 5.34. We use the R-algorithm to generate configurations. The finite temperature transition occurs at $\beta\approx5.28$

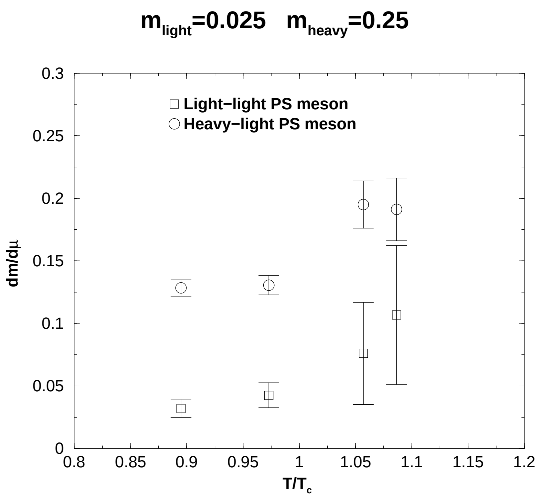

Figure 2. $\partial m/\partial\mu$ of light-light and heavy-light pseudoscalar mesons as a function of T/T_c.

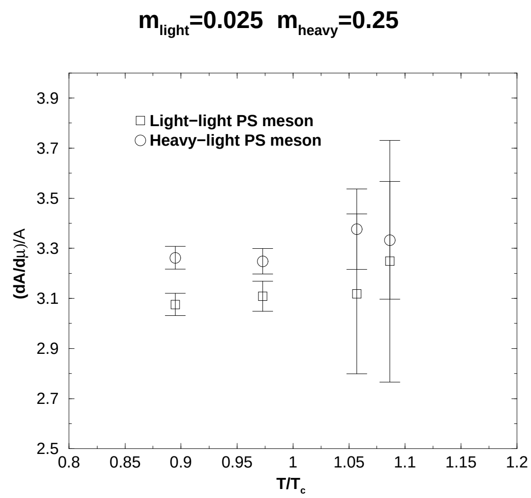

Figure 3. Response of the coupling A to chemical potential, $\partial \ln A/\partial\mu$ as a function of T/T_c.

[3] and the above β values are translated to $T/T_c = 0.90, 0.97, 1.06$ and 1.09 respectively.

We measure the pion screening mass. The quark propagator is calculated with $m_q = 0.025$ (light) and 0.25 (heavy). Then we construct the pion correlator with light-light and light-heavy quarks.

Fig. 1 shows the pion (light-light) correlation function $G_\pi(x)$ and its derivative with respect to μ at $\beta = 5.20$. We perform single pole fit for the data, which turned out to be sufficient for the pion channel.

Fig. 2 shows $\partial m/\partial\mu$ as a function of T/T_c. Despite the large errors we observe a systematic tendency towards raising the derivative of m above T_c.

Fig. 3 shows the response of the coupling A, $\partial \ln A/\partial\mu$ as a function of T/T_c. Both light-light and heavy-light mesons show similar values and no apreciable temperature dependence.

5. Discussions

Our preliminary results show remarkable characteristics of the response of meson masses to chemical potential. Possible interpretations for $\partial m/\partial\mu$ of the light-light system are as follows. The weak response of the mass below T_c indicates a persistence ofthe Nambu-Goldstone boson nature at least up to $T = 0.97T_c$. Growth of it above T_c is consistent with chiral restoration since the meson looses the Nambu-Goldstone character.

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